1

$P\,(3, -4)$ and $Q\,(-2, 0)$ are two points.

(a) Find the slope of the line PQ.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$x_1\ y_1$ $(3, -4)$ $x_2\ y_2$ $(-2, 0)$

$\dfrac{0 - \overset{+}{(-4)}}{-2 - 3} = \dfrac{4}{-5} = -\dfrac{4}{5}$ $m = -\dfrac{4}{5}$

(b) Find the equation of the line PQ.

$y - y_1 = m(x - x_1)$ $m = -\dfrac{4}{5}$ $x_1\ y_1$ $(3, -4)$

$y - (-4) = -\dfrac{4}{5}(x - 3)$

$5(y + 4) = -4(x - 3)$

$5y + 20 = -4x + 12 \longrightarrow 4x + 5y + 20 - 12 = 0 \rightarrow \boxed{4x + 5y + 8 = 0}$

(c) A line l passes through the point $(7, 5)$ and is parallel to PQ. Find the equation of l.

↑ So the slope is the same as PQ

$m = -\dfrac{4}{5}$

$x_1\ y_1$ $(7, 5)$

$y - y_1 = m(x - x_1)$

$y - 5 = -\dfrac{4}{5}(x - 7)$

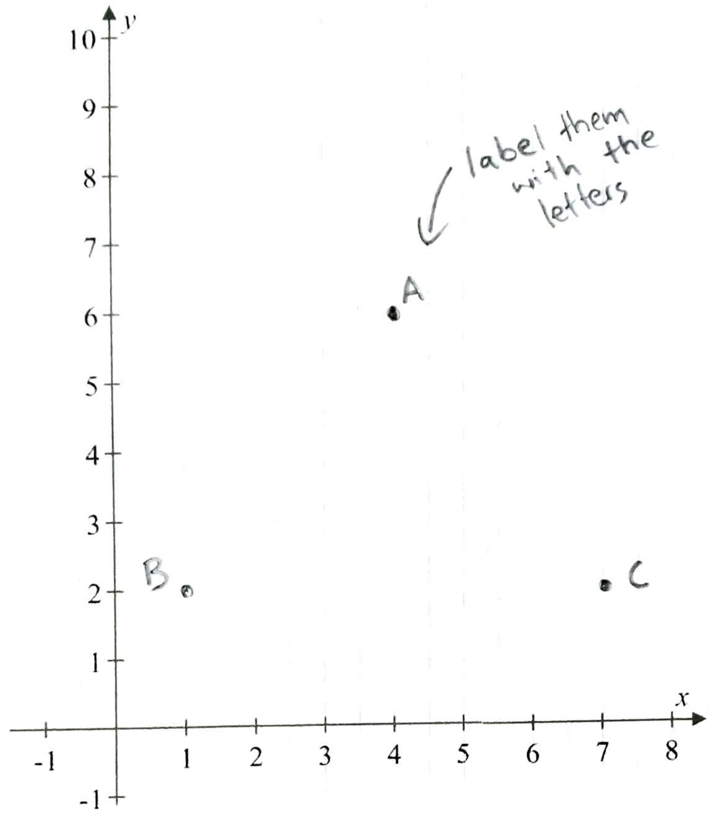
$5(y - 5) = -4(x - 7)$

$5y - 25 = -4x + 28$

$4x + 5y - 25 - 28 = 0$

Ans = $\underline{\underline{4x + 5y - 53 = 0}}$

(a) Plot the points $A\ (4, 6)$, $B\ (1, 2)$ and $C\ (7, 2)$ on the co-ordinate plane below. Label each point clearly.



label them with the letters

(b) Find the mid-point of [*BC*].

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \rightarrow \left(\frac{1 + 7}{2}, \frac{2 + 2}{2}\right)$$

$$B\ (\overset{x_1}{1}, \overset{y_1}{2}) \quad C\ (\overset{x_2}{7}, \overset{y_2}{2}) \qquad \left(\frac{8}{2}, \frac{4}{2}\right)$$

Ans = $(4, 2)$

(c) **(i)** Find $|BC|$, the distance from B to C.

Answer: $\overset{x_2}{7} - \overset{x_1}{1} = 6$ Ans = 6

(ii) Use the distance formula to find $|AB|$.

$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

A $(\overset{x_1}{4}, \overset{y_1}{6})$ B $(\overset{x_2}{1}, \overset{y_2}{2})$

$\sqrt{(1-4)^2 + (2-6)^2}$

$\sqrt{(-3)^2 + (-4)^2}$

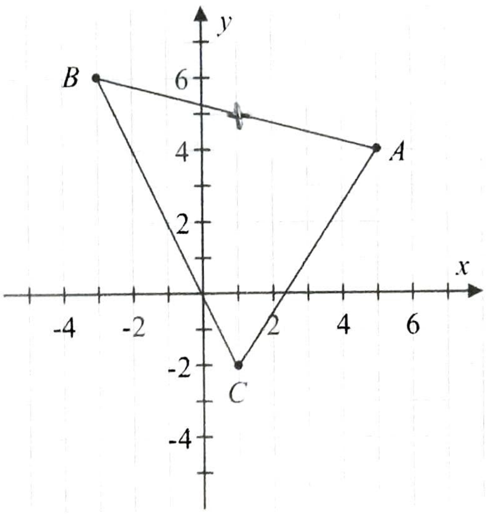
$\sqrt{9 + 16}$

$\sqrt{25}$

$|AB| = 5$

3

The diagram shows the triangle *ABC*.
The co-ordinates of the point *A* are (5, 4).



(a) Write down the co-ordinates of the points:

B $(-3, 6)$

C $(1, -2)$

(b) **(i)** On the diagram, mark the point *M*, the midpoint of [*AB*].

(ii) Use a formula to find the co-ordinates of *M*.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$AB \quad A\overset{x_1\,y_1}{(5,4)} \quad B\overset{x_2\,y_2}{(-3,6)}$$

$$\left(\frac{5+(-3)}{2}, \frac{4+6}{2}\right)$$

$$\left(\frac{5-3}{2}, \frac{10}{2}\right)$$

$$\left(\frac{2}{2}, \frac{10}{2}\right)$$

$$(1, 5)$$

(c) Use a formula to find the length of [*BC*], the longest side of the triangle.

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$|BC| \quad B\overset{x_1\,y_1}{(-3,6)} \quad C\overset{x_2\,y_2}{(1,-2)}$$

$$\sqrt{(1-(-3))^2+(-2-6)^2}$$

$$\sqrt{(1+3)^2+(-8)^2}$$

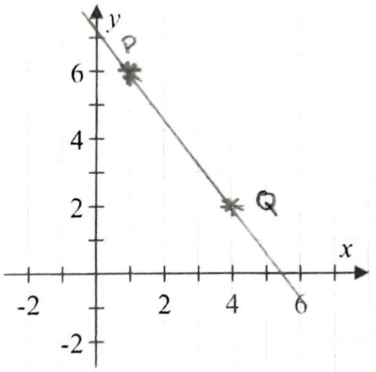
$$\sqrt{(4)^2+(-8)^2}$$

$$\sqrt{16+64}$$

$$\sqrt{80} \quad = 4\sqrt{5} \quad \text{or} \quad 8.9$$

The line l_1 passes through the points $P(1, 6)$ and $Q(4, 2)$.
The line l_2 has equation $3x - 4y - 4 = 0$.

(a) Plot P and Q on the diagram and show the line l_1. →



(b) Find the slope of the line l_1.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad \frac{2-6}{4-1}$

$P(\overset{x_1}{1}, \overset{y_1}{6})$ $Q(\overset{x_2}{4}, \overset{y_2}{2})$ $\qquad -\frac{4}{3}$

(c) State whether or not the two lines l_1 and l_2 are perpendicular to each other. Give a reason for your answer.

slope of $L_1 = -\frac{4}{3}$

slope of $L_2 = ?$

Equation of line of l_2 in standard form

$3x - 4y - 4 = 0$ ← let's rearrange it

$-4y = -3x + 4$

$y = \frac{-3}{-4}x + \frac{4}{-4}$

$y = \frac{3}{4}x - 1$

This is the slope of L_2

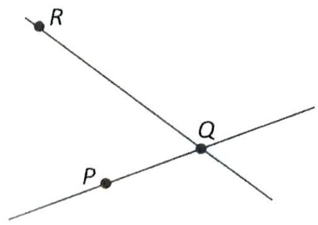
now we multiply the slopes. If we get -1 the two lines are perpendicular

$\overset{m \text{ of } l_1}{-\frac{4}{3}} \times \overset{m \text{ of } l_2}{\frac{3}{4}} = -1$

So $L_1 + L_2$ are perpendicular $\perp$

Question 2 **(25 marks)**

The diagram shows the line PQ and the line QR.
The co-ordinates of the points are $P(4, 2)$, $Q(8, 5)$ and $R(2, 11)$.



(a) Find the slope of PQ.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad \frac{5-2}{8-4} = \frac{3}{4}$

$\overset{x_1 \; y_1}{P(4,2)} \quad \overset{x_2 \; y_2}{Q(8,5)}$

m of $pq = \frac{3}{4}$

(b) Find the equation of the line PQ.
Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

Formula : $y - y_1 = m(x - x_1)$

$m = \frac{3}{4}$ $\qquad y - 2 = \frac{3}{4}(x - 4)$

$\overset{x_1 \; y_1}{P(4,2)}$ $\qquad 4(y-2) = 3(x-4)$

$4y - 8 = 3x - 12$

$-3x + 4y + 12 - 8 = 0$

Use only one set of Coordinates.

$\boxed{-3x + 4y + 4 = 0}$

Ans

or $3x - 4y - 4 = 0$

(c) Write down the slope of any line perpendicular to *PQ*.

Slope = $-\frac{4}{3}$

m of PQ $= \frac{3}{4} \leftrightarrows -\frac{4}{3}$

↗

flip it and change the sign

(d) Find the area of the triangle *PQR*.

$\frac{1}{2}|x_1y_2 - x_2y_1|$

1st we move one point to (0,0), then move the other points the same distance.

$(4,2)\ (8,5)\ (2,11)$

$-4 \downarrow -2 \quad -4 \downarrow -2 \quad -4 \downarrow -2$

$(0,0)\ (4,3)\ (-2,9)$

$(\overset{1}{4},\overset{2}{3})$

$\times$

$(-2,9)$

$\frac{1}{2}|(4)(9) - (3)(-2)|$

$\frac{1}{2}|36 - (-6)|$

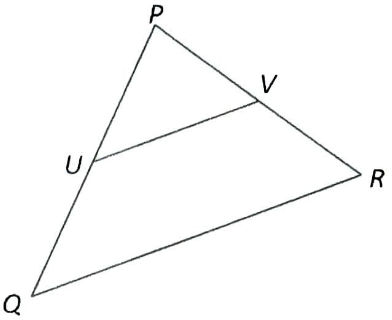
$\frac{1}{2}|36 + 6|$

$\frac{1}{2}|42|$

Ans: 21 units^2

Question 2 **(25 marks)**

The points $P(7, 10)$, $Q(1, 2)$ and $R(11, 4)$ are the vertices of the triangle shown.
The point $U(4, 6)$ is the midpoint of $[PQ]$ and the point V is the midpoint of $[PR]$.



(a) Find the co-ordinates of V.

V is the midpoint of P and R.

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ $\left(\frac{7+11}{2}, \frac{10+4}{2}\right)$

$P(\overset{x_1}{7}, \overset{y_1}{10})$ $R(\overset{x_2}{11}, \overset{y_2}{4})$ $V = (9, 7)$

(b) Show, by using slopes, that UV is parallel to QR.

* If two lines are parallel their slopes should be the same

1st UV $\frac{y_2-y_1}{x_2-x_1}$ $\frac{7-6}{9-4}$

$U(\overset{x_1}{4}, \overset{y_1}{6})$ $V(\overset{x_2}{9}, \overset{y_2}{7})$

$m = \frac{1}{5}$

2nd QR $\frac{y_2-y_1}{x_2-x_1}$ $\frac{4-2}{11-1}$

$Q(\overset{x_1}{1}, \overset{y_1}{2})$ $R(\overset{x_2}{11}, \overset{y_2}{4})$

$\frac{2}{10} = \frac{1}{5}$

m of $UV = \frac{1}{5}$ & m of $QR = \frac{1}{5}$

(c) Find the area of the triangle PQR.

$\frac{1}{2}|x_1y_2 - x_2y_1|$

We need to move the coordinates:

$\overset{P}{(7,10)}$ $\overset{Q}{(1,2)}$ $\overset{R}{(11,4)}$

$-1 \downarrow -2$ $-1 \downarrow -2$ $-1 \downarrow -2$

$(6,8)$ $(0,0)$ $(10,2)$

$(\overset{1}{6}, \overset{2}{8})$

$(10, 2)$

$\frac{1}{2}|(6)(2) - (8)(10)|$

$\frac{1}{2}|12 - 80|$

$\frac{1}{2}|-68| \longrightarrow \boxed{34 \text{ units}^2}$

we can ignore this minus

(d) The point S is the image of the point Q under the translation $\overrightarrow{UV}$.
Find the coordinates of S.

$U \rightarrow V$

$(4,6) \xrightarrow[+1]{+5} (9,7)$

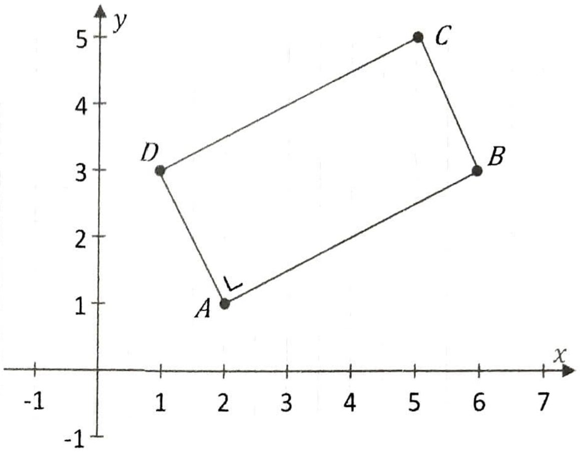
$Q \rightarrow S$

$(1,2) \xrightarrow[+1]{+5} (6,3)$

Question 3 **(25 marks)**

(a) The points $A(2,1)$, $B(6,3)$, $C(5,5)$, and $D(1,3)$ are the vertices of the rectangle $ABCD$ as shown.

(i) Show that $|AD| = \sqrt{5}$ units.



$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$A\,(\overset{x_1\,y_1}{2,1})\quad D\,(\overset{x_2\,y_2}{1,3})$

$\sqrt{(1-2)^2+(3-1)^2}$

$\sqrt{(-1)^2+(2)^2}$

$\sqrt{1+4}$

$\sqrt{5} = |AD|$

(ii) Find, in square units, the area of the rectangle $ABCD$.

In order to find the area we need to multiply the length by the width. We need to find $|AB|$ or $|DC|$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\quad A\,(\overset{x_1\,y_1}{2,1})\quad B\,(\overset{x_2\,y_2}{6,3})$

$\sqrt{(6-2)^2+(3-1)^2}$

$\sqrt{(4)^2+(2)^2}$

$\sqrt{16+4}$

$\sqrt{20}$

now

$\sqrt{20} \times \sqrt{5} = \sqrt{20 \times 5}$

↳ $\sqrt{100}$

Ans ↳ $\boxed{10 \text{ units}^2}$

(b) Find the equation of the line BC.
Give your answer in the form $ax + by + c = 0$, where a, b, and $c \in \mathbb{Z}$.

$y - y_1 = m(x - x_1)$

we need the slope and one set of coordinates

$\dfrac{y_2-y_1}{x_2-x_1}\qquad \dfrac{5-3}{5-6} = \dfrac{2}{-1} = -2$

$\underline{B(\overset{x_1\,y_1}{6,3})}\;C\,(\overset{x_2\,y_2}{5,5})$

let's use B

$y - 3 = -2(x - 6)$

$y - 3 = -2x + 12$

$2x + y - 3 - 12 = 0$

Ans $= \underline{\underline{2x + y - 15 = 0}}$

Question 8 **(40 marks)**

The diagram below shows the triangles *CBA* and *CDE*.

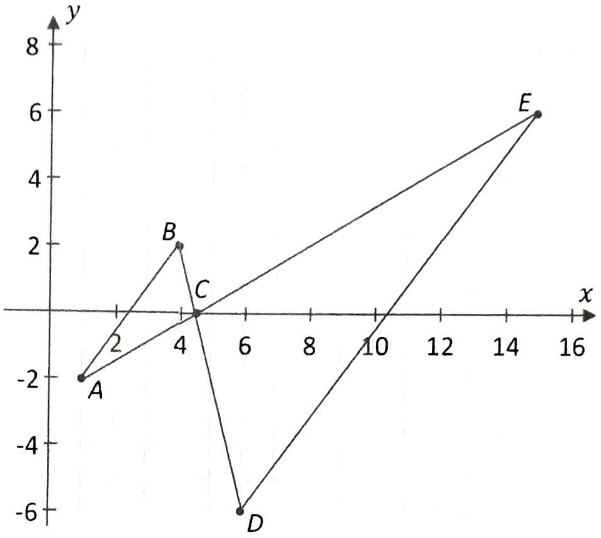
(a) The coordinates of *C* are $(4{\cdot}5, 0)$.
From the diagram, write down the coordinates of the points *A*, *B*, *D*, and *E*.

A = (1 , -2)

B = (4 , 2)

D = (6 , -6)

E =(15 , 6)



(b) Show, using slopes, that the line segments [*AB*] and [*DE*] are parallel.

Slope of AB $\frac{y_2-y_1}{x_2-x_1}$

$\overset{x_1\ y_1}{A(1,-2)}$ $\overset{x_2\ y_2}{B(4,2)}$

$\frac{2-(-2)}{4-1}$

$\frac{2+2}{3}$

$\frac{4}{3}$

Slope of DE $\frac{y_2-y_1}{x_2-x_1}$

$\overset{x_1\ y_1}{D(6,-6)}$ $\overset{x_2\ y_2}{E(15,6)}$

$\frac{6-\overset{+}{(-6)}}{15-6}$

$\frac{12}{9}$

$\frac{4}{3}$

$\therefore$ m of AB = m of DE

↑

This means therefore

(c) **(i)** Show that the area of the triangle *CBA* is 4 square units.

$\frac{1}{2}|x_1y_2 - x_2y_1|$

$C(4.5, 0)$ $B(4, 2)$ $A(1, -2)$

$-4.5 \downarrow 0$ $-4.5 \downarrow 0$ $-4.5 \downarrow 0$

$(0, 0)$ $(-0.5, 2)$ $(-3.5, -2)$

$\frac{1}{2}|(-0.5)(-2) - (2)(-3.5)|$

$(-0.5, 2)$

$(-3.5, -2)$

$\frac{1}{2}|1 + 7|$

$\frac{1}{2}|8|$

$\underline{\underline{4 \text{ units}^2}}$

(ii) Find $|AB|$, the distance from *A* to *B*.

$\overset{x_1\ y_1}{A(1, -2)}$ $\overset{x_2\ y_2}{B(4, 2)}$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(4-1)^2 + (2-(-2))^2}$

$\sqrt{(3)^2 + (4)^2}$

$\sqrt{9+16} \longrightarrow \sqrt{25} = \underline{\underline{5 \text{ units}}}.$

(iii) The triangle *CDE* is an enlargement of the triangle *CBA*.
Given that $|DE| = 15$ units, find the scale factor of the enlargement.

Scale factor =

(iv) Use this scale factor to find the area of the triangle *CDE*.

(a) The line l contains the points $A(4, 5)$ and $B(2, 0)$. Find the equation of l.
Give your answer in the form $ax + by + c = 0$ where a, b, and $c \in \mathbb{Z}$.

Slope $\frac{y_2-y_1}{x_2-x_1}$ $\frac{0-5}{2-4}$

$\overset{x_1\,y_1}{A(4,5)}\ \overset{x_2\,y_2}{B(2,0)}$

$\frac{-5}{-2}$

$\frac{5}{2}$

$y-y_1=m(x-x_1)$

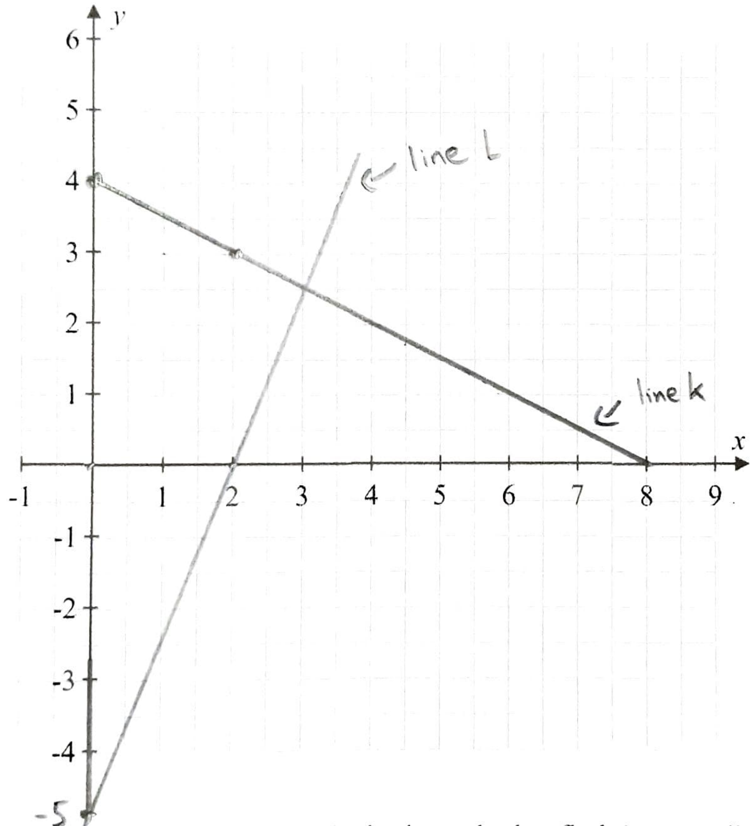
$y-5=\frac{5}{2}(x-4)$

$2(y-5)=5x-20$

$2y-10=5x-20$

$-5x+2y+20-10=0 \rightarrow \underline{-5x+2y+10=0}$

(b) Draw the line k: $x + 2y = 8$ on the axes below.



Method one :

$x+2y=8$

$2y=-x+8$

$y=-\frac{1}{2}x+\frac{8}{2}$

$y=-\frac{1}{2}x+4$

method two :

$x+2y=8$

let $x=0$	let $y=0$
$0+2y=8$	$x+2(0)=8$
$y=\frac{8}{2}$	$x=8$
$y=4$	
$(0,4)$	$(8,0)$

(c) Use a graphic, numeric or algebraic method to find the co-ordinates of $l \cap k$. ← This means intersects.

line L method 1

$-5x+2y+10=0$

$2y=5x-10$

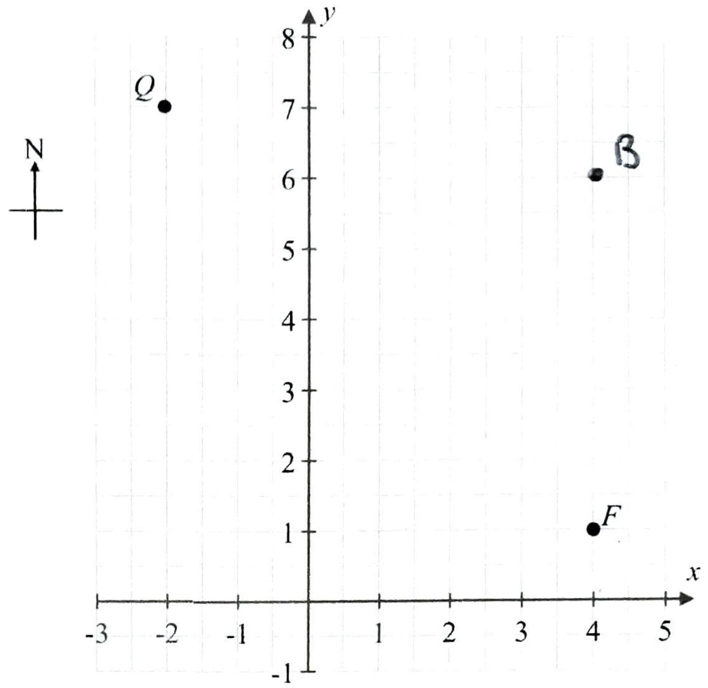
$y=\frac{5}{2}x-\frac{10}{2}$

$y=\frac{5}{2}x-5$

$-5x+2y=-10$

let $x=0$	let $y=0$
$-5(0)2y=-10$	$-5x+2(0)=-10$
$2y=-10$	$-5x=-10$
$y=\frac{-10}{2}$	$x=\frac{-10}{-5}$
$y=-5$	$x=2$
$(0,-5)$	$(2,0)$

Joe wants to draw a diagram of his farm. He uses axes and co-ordinates to plot his farmhouse at the point F on the diagram below.



(a) **(i)** Write down the co-ordinates of the point F.

$F = (4 , 1)$

(ii) A barn is 5 units directly North of the farmhouse. Plot the point representing the position of the barn on the diagram. Label this point B.

(b) Joe's quad bike is marked with the point Q on the diagram.
Find the distance from the barn (B) to the quad (Q).
Give your answer correct to 2 decimal places.

$\overset{x_1\ y_1}{B(4,6)}\ \overset{x_2\ y_2}{Q(-2,7)}$

$|BQ|\ \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(-2-4)^2+(7-6)^2}$

$\sqrt{(-6)^2+(1)^2}$

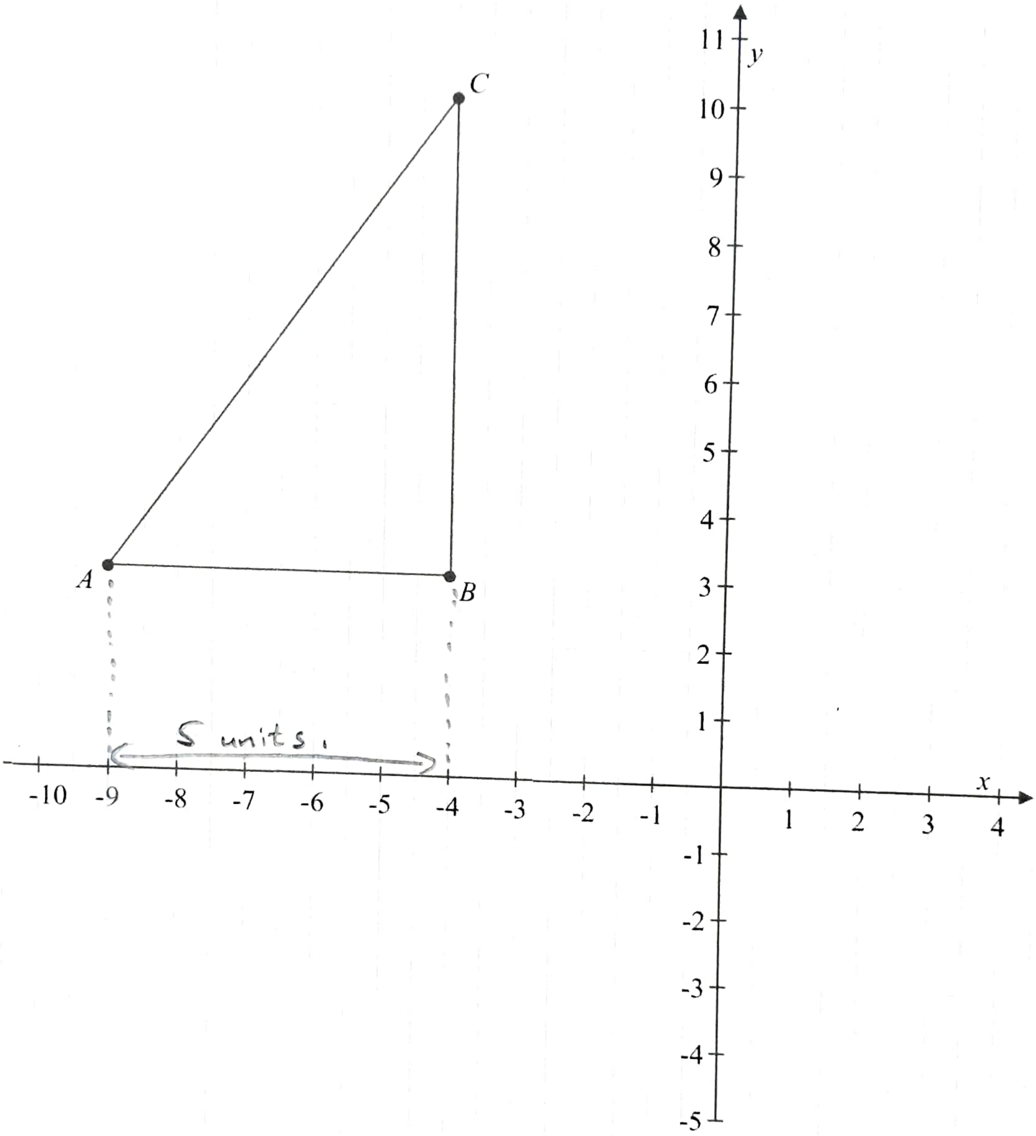
$\sqrt{37}$

Ans $= \underline{\underline{6 \cdot 08}}$

(c) Joe's tractor is at the point T, where $FBQT$ is a parallelogram.
Plot T on the diagram and write the co-ordinates of T in the space below.

$T = (\quad , \quad)$

The points $A(-9, 3)$, $B(-4, 3)$ and $C(-4, 10)$ are the vertices of the triangle ABC, as shown.



(a) **(i)** Find the length of $[AB]$.

AB A is above -9 on the x axis

B is above -4 on the x axis

so the difference between -9 and -4 is 5 units

so the length of |AB| is 5 units

(ii) Find the area of the triangle *ABC*.

$\frac{1}{2}|x_1y_2 - x_2y_1|$

$\frac{1}{2}|(-5)(7) - (0)(0)|$

$\frac{1}{2}|-35|$

Ans $\underline{\underline{17.5 \text{ units}^2}}$

A (-9,3) B (-4,3) C (-4,10)

$+4 \downarrow -3$ $+4 \downarrow -3$ $+4 \downarrow -3$

(-5,0) (0,0) (0,7)

$(\overset{1}{-5}, \overset{2}{0})$

$\times$

$(0,7)$

(b) *X*(2, −4) and *Y*(2, 1) are two points.

(i) Draw, on the diagram above, a triangle, *XYZ*, which is congruent to the triangle *ABC*.

(ii) Write down the co-ordinates of *Z* and explain why the triangle *XYZ* is congruent to the triangle *ABC*.

Z = (,)

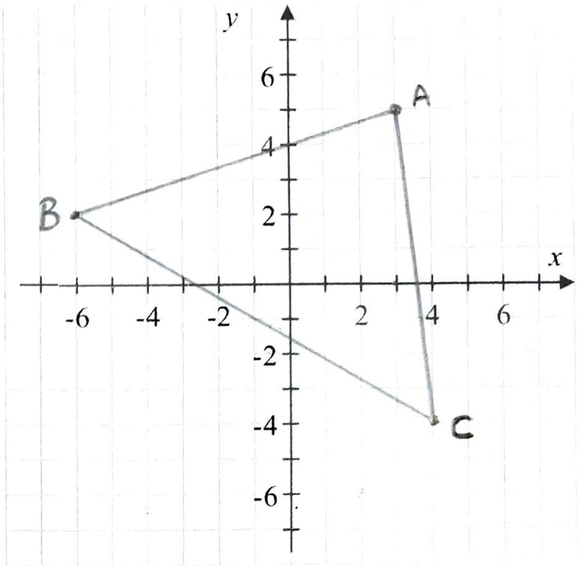
Reason:

Question 3 **(25 marks)**

The points *A*, *B*, and *C* have co-ordinates as follows:

A (3, 5)
B (−6, 2)
C (4, −4).

(a) Plot *A*, *B*, and *C* on the diagram.



(b) Find the equation of the line *AB*.

$\frac{y_2-y_1}{x_2-x_1}$ A $(\overset{x_1}{3},\overset{y_1}{5})$ B $(\overset{x_2}{-6},\overset{y_2}{2})$

$\frac{2-5}{-6-3} = \frac{-3}{-9} = \frac{1}{3}$

A $(\overset{x_1}{3},\overset{y_1}{5})$

$y-5=\frac{1}{3}(x-3)$

$3(y-5)=1(x-3)$

$3y-15=1x-3$

$-x+3y-15+3=0$

Ans $\boxed{-x+3y-12=0}$

(c) Find the area of the triangle *ABC*.

$\frac{1}{2}|x_1y_2-x_2y_1|$

$\frac{1}{2}|(9)(-6)-(3)(10)|$

$\frac{1}{2}|-54-30|$

$\frac{1}{2}|-84|$

Ans: $\underline{42 \text{ units}^2}$

A (3,5) B(−6,2) C (4,−4)

$+6 \downarrow -2$ $+6 \downarrow -2$ $+6 \downarrow -2$

(9,3) (0,0) (10,−6)

$(\overset{1}{9},\overset{2}{3})$

$\times$

$(10,-6)$

(a) l is the line $3x + 2y + 18 = 0$. Find the slope of l.

$$3x + 2y + 18 = 0$$
$$2y = -3x - 18$$
$$y = -\frac{3}{2}x - \frac{18}{2} \longrightarrow y = -\frac{3}{2}x - 9 \qquad m = -\frac{3}{2}$$

This is the slope

(b) The line k is perpendicular to l and cuts the x-axis at the point $(7, 0)$.
Find the equation of k.

m of $l = -\frac{3}{2}$

if line k is ⊥ then we can flip the slope of line l and change the sign.

m of $k = \frac{2}{3}$

$$y - 0 = \frac{2}{3}(x - 7)$$
$$3(y - 0) = 2(x - 7)$$

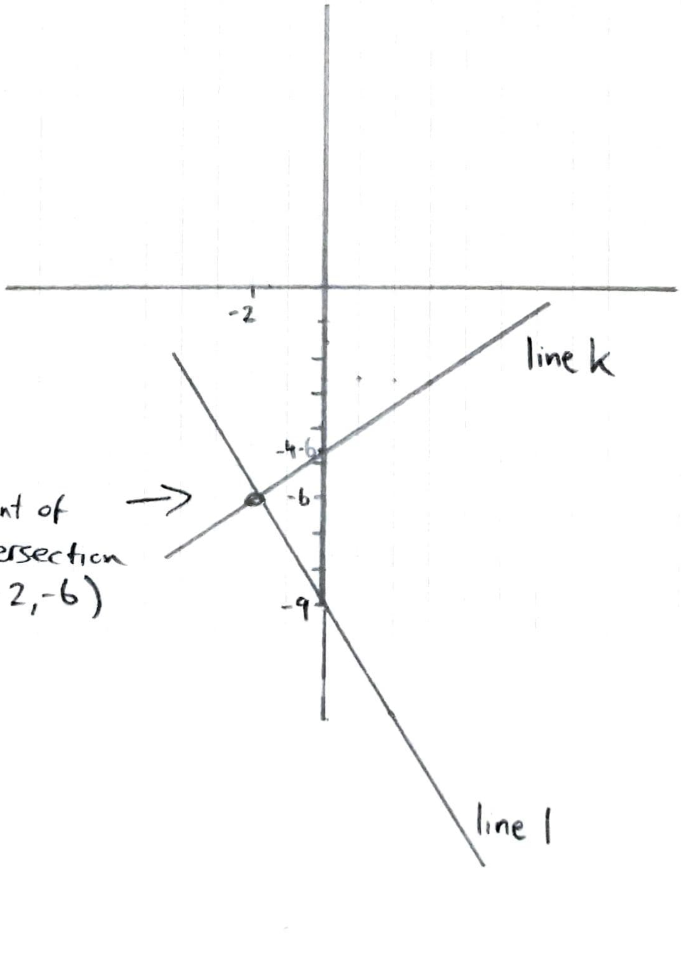
and using the point (7,0) we fill out the formula

$$3y = 2x - 14 \longrightarrow \boxed{-2x + 3y + 14 = 0}$$

(c) Find the co-ordinates of the point of intersection of the lines l and k.

line l $= y = -\frac{3}{2}x - 9$

line k
$$-2x + 3y + 14 = 0$$
$$3y = 2x - 14$$
$$y = \frac{2}{3}x - \frac{14}{3}$$
$$y = \frac{2}{3}x - 4.6$$



Point of intersection (-2,-6)

2. **(a)** Find the area of the triangle with vertices (0, 0), (8, −6) and (−1, 5).

(b) *l* is the line $3x-4y-15=0$.

(i) Verify that $(1,-3)$ is a point on *l*.

(ii) *l* intersects the *x*-axis at *P*. Find the co-ordinates of *P*.

The line *k* passes through the point $(1,-3)$ and is perpendicular to *l*.

(iii) Show the lines *l* and *k* on a co-ordinate diagram.

(iv) Find the equation of *k*.

(c) *A*(2, −1) and *B*(−4, 7) are two points.

(i) Find | *AB* |.

(ii) Find *C*, the image of *B* under the translation $(2,-1)\rightarrow(-7,11)$.

(iii) Show that | *AB* | : | *AC* | = 2 : 5.

2 (a) $\frac{1}{2}\left|x_1y_2-x_2y_1\right|$

$(\overset{1}{8},\overset{2}{-6})$

$\times$

$(-1,5)$

$\frac{1}{2}\left|(8)(5)-(-6)(-1)\right|$

$\frac{1}{2}\left|40-6\right|$

$\frac{1}{2}\left|34\right|$

17 units^2

2. **(a)** Find the area of the triangle with vertices (0, 0), (8, −6) and (−1, 5).

(b) l is the line $3x - 4y - 15 = 0$.

(i) Verify that $(1, -3)$ is a point on l.

(ii) l intersects the x-axis at P. Find the co-ordinates of P.

The line k passes through the point $(1, -3)$ and is perpendicular to l.

(iii) Show the lines l and k on a co-ordinate diagram.

(iv) Find the equation of k.

~~**(c)**~~ $A(2, -1)$ and $B(-4, 7)$ are two points.

~~**(i)**~~ Find $|AB|$.

(ii) Find C, the image of B under the translation $(2, -1) \rightarrow (-7, 11)$.

(iii) Show that $|AB| : |AC| = 2 : 5$.

2 (b) (i) $3x - 4y - 15 = 0$

$3(1) - 4(-3) - 15 = 0$ (1,−3) is on the line l.

$3 + 12 - 15 = 0$

$15 - 15 = 0$

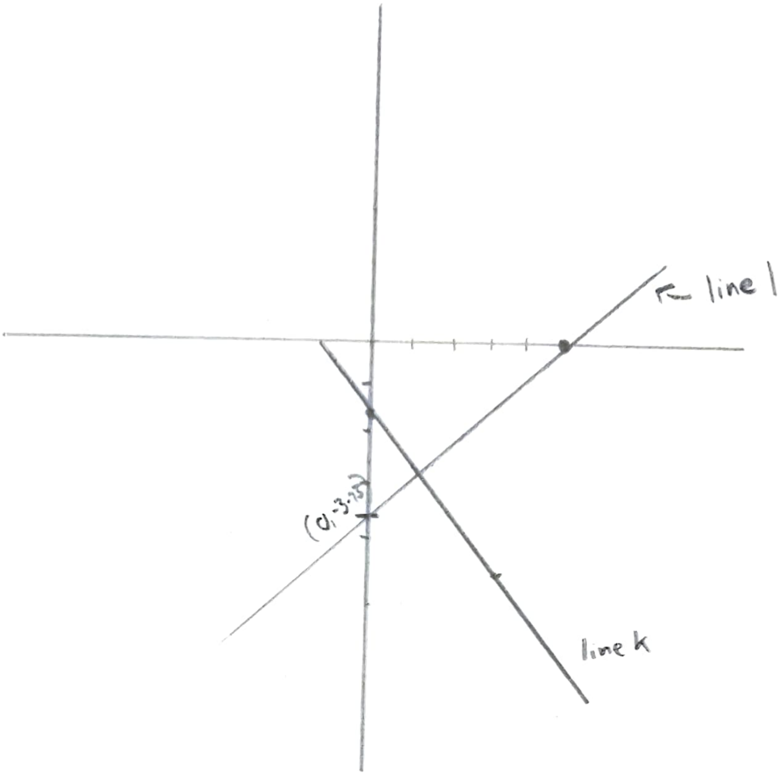
$0 = 0$

(ii) $3x - 4y = 15$

let $x = 0$	let $y = 0$
$3(0) - 4y = 15$	$3x - 4(0) = 15$
$-4y = 15$	$3x = 15$
$y = \frac{15}{-4}$	$x = \frac{15}{3}$
$y = -3.75$	$x = 5$

p: (5,0) is where l intersects the x axis

2 (b) (iii)



(iv) $l = 3x - 4y - 15 = 0$

$-4y = -3x + 15$

$y = \frac{-3}{-4}x + \frac{15}{-4}$

$y = \underset{\uparrow m}{\frac{3}{4}}x - 3.75$

$k = ?$

m of k $= -\frac{4}{3}$ and we have the point $(1,-3)$

$y-(-3) = -\frac{4}{3}(x-1)$

$3(y+3) = -4(x-1)$

$3y+9 = -4x+4$

$3y = -4x+4-9$

$3y = -4x-5$

$y = -\frac{4}{3}x - \frac{5}{3}$